Application of Fuzzy Logic to System Reliability

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Abstract

Structure and discipline during the conceptual design phase are fundamental to the efficient and effective development of competitive products and systems. However, during this phase the extent and resolution of information available regarding system parameters such as reliability is typically characterized by imprecision and subjectivity. Such uncertainty can be represented by fuzzy variables and linguistic values. Discrimination between competing product concepts thus necessitates identification of a defuzzification technique to allow comparison of imprecise requirements with imprecise expected values for conceptual alternatives. A defuzzification technique based on the degree of compliance between required and anticipated parameter values, well suited to concept evaluation and selection, is presented. The advantage of such an approach is demonstrated and a fuzzy logic based framework for this purpose is proposed. Finally, techniques for accelerating such a compliance analysis are suggested.

1. Introduction

In many system design and development scenarios of practical interest, it is necessary to combine multiple information sources. This combined 'measure' is then used to select the optimal conceptual design from a number of alternatives. However, the information available during this period is typically characterized by imprecision and subjectivity. This type of uncertainty is well represented by linguistic values and fuzzy variables (Guth (1991), Park et al (1990)). Fuzzy sets have been successfully applied to a wide variety of different fields, including pattern recognition and classification, robotics, information processing and communication (Bazu (19921)). A relatively recent application is to the selection of the optimal conceptual system design from a number of competing alternatives. In order to be of practical value, subsequent to selection the fuzzy results must be converted to crisp or precise numerical values: so-called defuzzification. Choice of an appropriate technique is fundamental to a successful conceptual design process. The two most commonly approaches to defuzzification will now be presented, and the advantages of the compliance analysis approach in the context of alternative conceptual design evaluation will be argued. The practical application of this technique will be illustrated through an example.

1.1 Separation Analysis Defuzzification Techniques

Classical separation analysis, or 'rating and ranking' defuzzification methods essentially fall into two broad categories, either based on the notion of mapping every fuzzy subset (each representing a different alternative) onto the real number line, where natural and total order exists, or on the development of a fuzzy set containing all the alternatives (together with their corresponding membership function values, which indicate the degree to which each alternative may be considered best) as elements. Within each of these categories, there are a variety of separation analysis techniques. For a comprehensive discussion of fuzzy subset ranking methods, see Runkler *et al.* (1993), Verma (1994a), Bortolan *et al*. (1985), Roubens (1986) or Li *et al*. (1987). These methods were compared by Bortolan and Degani (1985) on a group of selected examples. They discovered that there is generally a lack of discrimination between alternatives, and this assertion was confirmed by Baldwin and Guild (1979). In addition, the results generated by these methods are occasionally inconsistent or conflict with intuition and often require a large computational effort.

1.2 Compliance Analysis Defuzzification Techniques

One motivation for the present work is to develop a technique for achieving optimal differentiation between alternative conceptual designs. In order to achieve this, it is necessary to first identify those design dependent parameters (DDP's) considered critical by both customer and designer. For each candidate design solution and every DDP, the predicted value of that DDP under the given design must be calculated. This is then compared with the required value of that DDP, obtained by correlating customer and design requirements. Due to the extent and resolution of available information, this comparison often reduces to evaluating and comparing imprecise or fuzzy values. Thus, use of any of the standard separation analysis techniques is probably not the most suitable approach because, subsequent to defuzzification and selection of a 'best' design alternative (for each DDP), the chosen design alternative still has to be assessed with regard to its compliance with customer and design requirements (themselves often specified in fuzzy terms). A considerably better approach is through the use of a compliance analysis, in which the feasibility of every candidate design alternative (for each DDP) is assessed as an integral part of defuzzification procedure. In this approach, the degree of compliance between the (fuzzy) required and predicted values is used to differentiate between alternative conceptual designs. Thus, considerable research effort has recently been directed towards developing improved compliance analysis defuzzification methods. The fuzzy weighted wedge mechanism developed by Verma *et al*. (1996) has demonstrated significant improvement over existing procedures. It will be introduced in the following section, together with an example of its application to system reliability analysis.

2. Fuzzy Weighted Wedge Mechanism

When comparing alternative conceptual designs, system-level DDP requirements and predictions can be translated into fuzzy algebraic expressions according to the fuzzy weighted wedge methodology developed by Verma (1994a). Both numerical and non-numerical DDP's can be represented in this manner. A triangular or trapezoidal shape is assumed, although the methodology is independent of the shape of the DDP profiles, provided they are all normal and convex. For every DDP preference (requirement) profile that is produced, one anticipation (prediction) profile for each conceptual design alternative is also constructed. Evaluation and comparison of competing designs then reduces to a comparative analysis of pairs of fuzzy profiles; so-called Feasibility Assessment (FA). This is achieved through construction of a Feasibility Index (FI), which is a non-fuzzy indication of the degree of overlap between them. The FI takes into account the associated preference level (location of overlap) as well as its magnitude (area) using a weighting wedge. For example, consider the design dependent parameter '*reliability*'. This may be expressed in fuzzy terms as, for example, '*greater than 40,000 hours MTBF'*. Such a requirement can be stated algebraically as

$$y = \begin{cases} 0 & x \leq 36{,}000 \\ \dfrac{x}{4000} - 9 & 36{,}000 \leq x \leq 40{,}000 \\ 1 & x \geq 40{,}000 \end{cases}$$

or shown graphically as

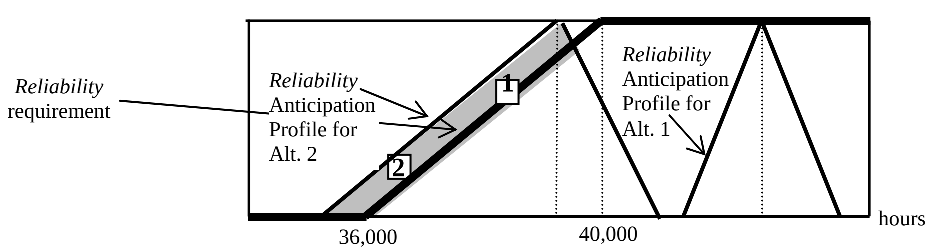

Figure 1: Feasibility assessment of fuzzy requirement and anticipation profiles for DDP 'reliability'

Figure 1 shows the preference profile for the particular DDP *reliability*, together with the anticipation (prediction) profiles for two alternative conceptual designs, 1 and 2. As stated above, feasibility analysis of potential design concepts comprises a comparison of fuzzy profiles representing required and predicted/estimated values of DDP's. The 'non-overlap' area suggests the lack of compliance between the two. However from a designer's perspective, a unit area of non-overlap at higher preference levels, such as Area 1 in Figure 1, is less desirable than a unit area of non-overlap at lower preference levels, such as Area 2. One method of accounting for the importance of different areas of non-overlap is by means of a weighting wedge as shown in Figure 2. The area of non-overlap between the two profiles in the original vertical plane is then projected onto a plane inclined at some angle to this vertical plane. This process captures a volume, which facilitates the comparison of different regions of non-overlap based upon both area and associated preference level. The weighting wedge mechanism allows to the computation of a Feasibility Index (FI) value for each *concept:DDP* combination. For example, the feasibility index for the DDP *reliability*, for the *n*th design concept, $FI_{(reliability,n)}$, is defined as the ratio of the projected overlap volume to the total projected volume. The feasibility index is a (non-fuzzy) number between zero and one. The closer the FI is to one, for any design concept and a given DDP, the better that particular conceptual design meets that DDP requirement.

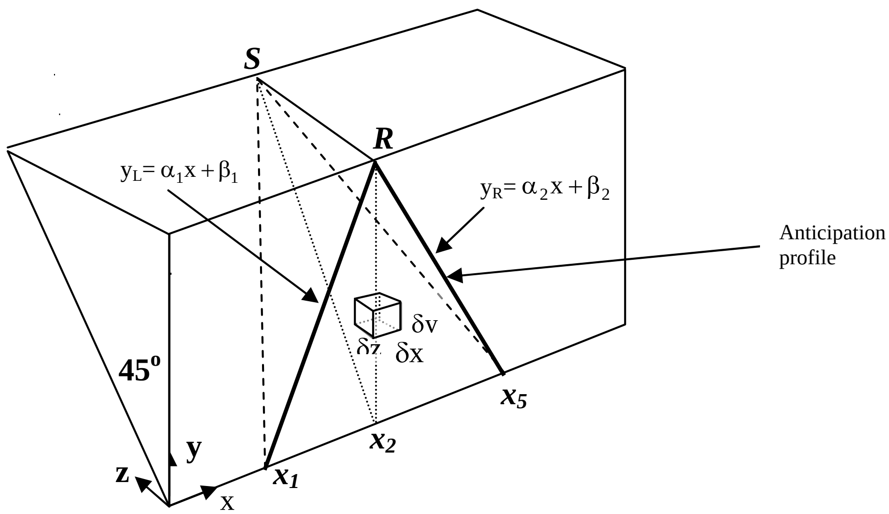

Figure 2: Volumetric analysis of overlap/non-overlap between preference and anticipation profiles

The calculation of the overlap and total volumes projected onto the weighted wedge can be achieved in a number of ways. The simplest approach is to use a numerical approximation such as that of Verma & Knezevic (1996). However, an alternative recently developed (Verma *et al.* 1999) allows the exact evaluation of these projected volumes through calculation of triple volume integrals. The feasibility index associated with each DDP, for every alternative conceptual design, can be calculated. The overall merit for each feasible design concept is then computed by consolidating these values with the relevant DDP relative priorities. Overall merit is a unitless fuzzy number and its imprecision is a function of the uncertainty associated with the inputs. These overall merit profiles represent relative goodness, facilitate focused design iteration, and guide commitment to a preferred concept. After reviewing the analysis and evaluation results, the design team may decide to utilize the knowledge gained as a basis for revisiting the conceptual designs. The fuzzy weighted wedge compliance analysis may be accelerated by replacing the linear projection plane by a non-linear weighted wedge. Such a projection plane serves as a sensitivity mechanism to exaggerate the difference between feasible design alternatives, thereby facilitating the design decision making process. Non-linear planes also allow the definition of feasibility thresholds, and can be tailored for different applications.

3. Conclusions

A disciplined design process is essential for effective and efficient development of systems which are both responsive to customer needs and globally competitive. This paper presents a methodology for the systematic analysis, evaluation and selection of a design concept from a number of competing design alternatives. The paper first discusses the application of defuzzification techniques to conceptual design differentiation, and argues that the class of methods known as compliance analyses are most suited to this application. An extension to such techniques, known as the fuzzy weighted wedge mechanism, is presented, and its use for alternative design differentiation demonstrated. Techniques for accelerating this type of analysis were suggested.

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